647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2023 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any one. (07)

1. In usual notation prove: $\lim_{n\to\infty} \sqrt[n]{a} = 1$ if a > 0.
2. If $\lim_{n\to\infty} a_n = l$ then $\lim_{n\to\infty} \left(\frac{a_1 + a_2 + \cdots\ldots + a_n}{n}\right) = l$.

Que.1(B) Attempt any one. (04)

1. Discuss the convergence of the sequence $s_1 = \sqrt{16}$, $s_{n+1} = \sqrt{6s_n}$, n∈N, and Find its limit if exists.
2. Show that $\{S_n\}$ is divergent where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

Que.1(C) Attempt any one. (03)

1. Show that $\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] = 0$
2. Show that every convergent sequence is bounded.

Que.2(A) Attempt any one. (07)

1. Discuss the convergence of the series $1 + \frac{1}{2} + \frac{1\cdot 3}{2\cdot 4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6} + \cdots$
2. Discuss the convergence of $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} \ldots..$

Que.2(B) Attempt any one. (04)

1. Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$
2. Discuss the convergence of the series $1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots$

Que.2(C) Attempt any one. (03)

1. Find the radius of convergence for the series $\sum_0^\infty \frac{x^n}{n+2}$
2. Test the series $\sum sin\ \frac{1}{n}$

Que.3(A) Attempt any one. (07)

1. If $\bar{f}$ and $\bar{g}$ are irrotational function on D then prove that $\bar{f} \times \bar{g}$ is a solenoidal function.
2. Prove: Curl ($\varphi\bar{f}$) =φ curl $\bar{f}$ + grad $\varphi \times \bar{f}$.

Que.3(B) Attempt any one. (04)

1. Prove: $\nabla^2(log\ r) = \frac{1}{r^2}$ where r = $|\bar{r}| = \sqrt{x^2 + y^2 + z^2}$
2. If $\bar{f} = (x^3, y^3, z^3)$ then prove that Curl $\bar{f} = \bar{0}$ and grad (div $\bar{f}$) = $6\bar{r}$

Que.3(C) Attempt any one. (03)

1. If a vector function $\bar{F} = (x^3z - axyz)\ \hat{\imath}$ + (xy - $3x^2\ yz)\hat{\jmath}$ + (yz$^2 - xz)\hat{k}$ is solenoidal then find the value of a.
2. Prove: Curl ($\bar{r} \times \bar{a}$) = - $2\bar{a}$ where $\bar{a}$ is a constant vector.

Que.4(A) Attempt any one. (07)

1. Prove: $\iiint_R (x^2 + y^2 + z^2)^5$ dx dy dz = $\frac{4\pi a^{13}}{13}$ where R is $x^2 + y^2 + z^2 = a^2$.
2. Prove: $\iint_R (1 - x - y)^3 \sqrt{x}\sqrt{y}$ dx dy = $\frac{\pi}{480}$ Where R is triangle whose vertices are (0,0), (0,1) and (1,0).

Que.4(B) Attempt any one. (04)

1. Find the volume of a sphere having radius r.
2. Find $\iint_R xy\, dy\, dx$ where $R = \{(x,y)/\, x^2 \le y \le \sqrt{x}\,, 0 \le x \le 1\}$

Que.4(C) Attempt any one. (03)

1. Change the order of integral $\int_0^1 \int_{x^2}^1 f(x,y)dxdy$
2. Change in spherical co-ordinate in $\iiint_R x^2y\, dxdydz$, Where R is $x^2 + y^2 + z^2 \le a^2$

Que.5(A) Attempt any one. (07)

1. State and prove Green's theorem.
2. In usual notation derive the relation between beta and gamma function.

Que.5(B) Attempt any one. (04)

1. Find: $\iint_S x^2dydz + y^2dzdx + 2z(xy - x - y)dxdy$, Where $S{:}\, 0 \le x, y, z \le 1$.
2. Prove: $\int_0^\infty \frac{1}{1+x^4}\, dx = \frac{\pi}{2\sqrt{2}}$

Que.5(C) Attempt any one. (03)

1. Prove that for any vector field V, $\iint_S (curl\, V).n\, d\sigma = 0$.
2. Prove: $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \sqrt{2}\,\pi$.

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